

An Internal Variable Constitutive Model for the Large
Deformation of Metals at High Temperatures

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Elevated temperature deformation processing, "hot working," is central to the production of more than 80% of all metal products. The advent of large deformation finite element methodologies is beginning to permit the numerical simulation of hot working processes whose design until recently has been based on prior industrial experience. Proper application of such finite element techniques requires realistic constitutive equations which more accurately model material behavior during hot working.

The constitutive equations should satisfy several requirements. First, the equations should be able to model large, three dimensional deformations. Second, the model should encompass the range of hot working conditions, which includes strain rates ranging from 10^{-3} to 10^1 sec^{-1} or greater, homologous temperatures from .5 to .9, and interrupted deformation histories. Third, determination of parameters associated with the model should be straightforward and require a minimum of materials testing. Fourth, the model should provide a means of representing material microstructural state and its evolution during deformation. Finally, the model should be formulated with due consideration of issues regarding their numerical implementation in finite element programs.

A simple constitutive model for hot working which satisfies most of these requirements is the single-scalar internal variable model for isotropic thermo-elasto-viscoplasticity proposed by Anand [1985,1982]. In the next section we recall this constitutive model. The specific scalar functions for the equivalent plastic strain rate and the evolution equation for the internal variable presented here are slight modifications of those proposed by Anand [1982]. These modified functions are better able to represent high temperature material behavior. Following this presentation of the constitutive equations we briefly describe

our monotonic constant true strain rate and strain rate jump compression experiments on a 2% silicon iron. The material parameters appearing in the constitutive model can be determined from the stress-strain curves resulting from such experiments. The model is implemented in the general purpose finite element program ABAQUS [Hibbitt, et.al., 1984]. Finally, using this program, the predictive capabilities of the model are evaluated for some simple deformation histories.

Constitutive Model

(a) Stress-strain-temperature rate relation:

$$\dot{\tilde{\mathbf{T}}}^\nabla = \mathcal{L}[\mathbf{D} - \mathbf{D}^p] - \Pi \dot{\theta}, \quad (1)$$

where with \mathbf{T} denoting the Cauchy stress and \mathbf{F} denoting the deformation gradient,

$\tilde{\mathbf{T}} \equiv (\det \mathbf{F}) \mathbf{T}$	Kirchhoff stress;
$\tilde{\mathbf{T}}^\nabla \equiv \dot{\tilde{\mathbf{T}}} - \mathbf{W} \tilde{\mathbf{T}} + \tilde{\mathbf{T}} \mathbf{W}$	Jaumann derivative of Kirchhoff stress;
$\mathcal{L} = 2\mu \mathbf{I} + [\kappa - (2/3)\mu] \mathbf{1} \otimes \mathbf{1}$	Elasticity tensor;
$\mu = \hat{\mu}(\theta), \quad \kappa = \hat{\kappa}(\theta)$	Shear and bulk moduli;
$\Pi = (3\kappa\alpha) \mathbf{1}$	Stress temperature tensor;
α	Coefficient of thermal expansion;
$\mathbf{D} = \text{sym } \dot{\mathbf{F}} \mathbf{F}^{-1}$	Stretching tensor;
$\mathbf{W} = \text{skew } \dot{\mathbf{F}} \mathbf{F}^{-1}$	Spin tensor;
θ	Absolute temperature.

(b) Flow rule:

The constitutive equation for \mathbf{D}^p is:

$$\mathbf{D}^p = \dot{\bar{\epsilon}}^p \left(\frac{3}{2} \frac{\tilde{\mathbf{T}}'}{\bar{\sigma}} \right), \quad (2)$$

where

$$\begin{aligned} \dot{\bar{\epsilon}}^p &= f(\bar{\sigma}, \theta, s) > 0, \quad \bar{\sigma} < s && \text{equivalent plastic tensile strain rate,} \\ \bar{\sigma} &= \sqrt{(3/2) \tilde{\mathbf{T}}' \cdot \tilde{\mathbf{T}}'} && \text{equivalent tensile stress,} \end{aligned}$$

and s is a scalar internal variable with the dimension of stress. We call it the deformation resistance which may be associated with a microstructural characteristic such as dislocation density.

(c) Evolution equation:

The deformation resistance s is assumed to evolve according to:

$$\dot{s} = h(\bar{\sigma}, \theta, s) \dot{\bar{\epsilon}}^p - \dot{r}(\theta, s),$$

where h represents strain hardening (with dynamic recovery) and \dot{r} represents static recovery.

The major task in completing the constitutive model involves a specification of the rate equation f above and the evolution equation for the internal variable s . We propose the following model:

$$\dot{\bar{\epsilon}}^p = A \exp\left(-\frac{Q}{k\theta}\right) [\sinh(\xi \frac{\bar{\sigma}}{s})]^{1/m}, \quad \bar{\sigma} < s, \quad (3)$$

$$\dot{s} = h_0 [1 - (s/s^*)]^a, \quad a \geq 1, \quad (4)$$

with

$$s^* = \bar{s} \left[\frac{\dot{\bar{\epsilon}}^p}{A} \exp\left(\frac{Q}{k\theta}\right) \right]^n. \quad (5)$$

In this model the material parameters are $A, Q, \xi, m, h_0, a, \bar{s}$, and n , and k is Boltzmann's constant.

The static recovery function \dot{r} in this model is set equal to zero. Accordingly, these equations are unable to model recovery during hold periods between high rate deformation passes or during very slow deformation processing.

Compression Experiments on a 2% Silicon Iron

Isothermal hot-compression tests have been performed to evaluate the material constants associated in the above model. A 2% silicon iron was selected since it does not dynamically recrystallize and since it does not experience a change in crystal structure

upon quenching from elevated temperature deformation. Tests were performed on a high temperature materials test system consisting of a vacuum furnace mounted in a servo-hydraulic test machine. An analog function generator was used to produce constant true strain rates and strain rate jumps. Data was collected on a microcomputer data acquisition system.

The test specimens were circular cylinders with a height-to-diameter ratio 1.5. Shallow, concentric, circular grooves were machined into the ends of each specimen to hold a high temperature lubricant. Lubricants were mixtures of powdered glass and boron nitride powder. Homogeneous compression to a true strain of approximately -1.0 was achievable on a routine basis.

Two sets of experiments were performed: isothermal, constant true strain rate tests and strain rate jump tests. The constant true strain rate tests were performed for a range of temperatures from 800 to 1200 degrees Celsius, and a range of strain rates from 10^{-3} to 10^0 per second. Figure 1 provides a representative subset of these tests for a given temperature at varying true strain rates. Strain rate jump tests were performed to provide means of evaluating strain rate dependence *at a given internal state*, because such a test instantaneously decouples the strain rate equation (3) from the evolution equation for the internal variable (4). Figure 2 shows a representative series of jump tests at a constant temperature. The stress/strain data following the jump in strain rate also provides data for an independent comparison of the constitutive model predictions and actual material response.

Material Constant Determination and Model Evaluation

Material constants for the model represented by equations (3) to (5) have been determined using data obtained from the constant true strain rate and strain rate jump tests. The equations have been incorporated using procedures outlined by Anand [1986] via a user-material interface in the finite element program ABAQUS. Stress-strain curves

have been calculated using ABAQUS for conditions representative of the experiments on the silicon iron. The calculated curves are compared with the experimental data in Figures 3 and 4. The agreement between the theory and experiment is excellent.

Several test specimen geometries have also been evaluated which produce a gradient of internal microstructure within a single deformed specimen. These specimens have been deformed at temperature and then quenched. Different measures of microstructure, such as microhardness and etchpit density, have been compared with the variation in the internal variable predicted by the constitutive model.

References

- 1982 Anand, L., "Constitutive Equations for the Rate-Dependent Deformation of Metals at Elevated Temperatures," *Journal of Engineering Materials and Technology*, 104, January, 1982, pp. 12-17.
- 1984 H. D. Hibbitt, ABAQUS/EPGEN, "A General Purpose Finite Element Code with Emphasis on Non-Linear Applications," *Nuclear Engineering and Design* 77, 1984, pp 271-297. Also: ABAQUS manuals by Hibbitt, Karlsson, and Sorensen, Inc., Providence, R.I. (1984).
- 1985 Anand, L., "Constitutive Equations for the Hot Working of Metals," *International Journal of Plasticity*, 1, 1985, pp. 213-231.
- 1986 Anand, L., Lush, A., Briceno, M., and Parks, D., "A Time-Integration Procedure for a Set of Internal Variable Type Elasto-Viscoplastic Constitutive Equations," *Journal of Computers and Structures*, forthcoming.

Fe - 2% Si

1100 C

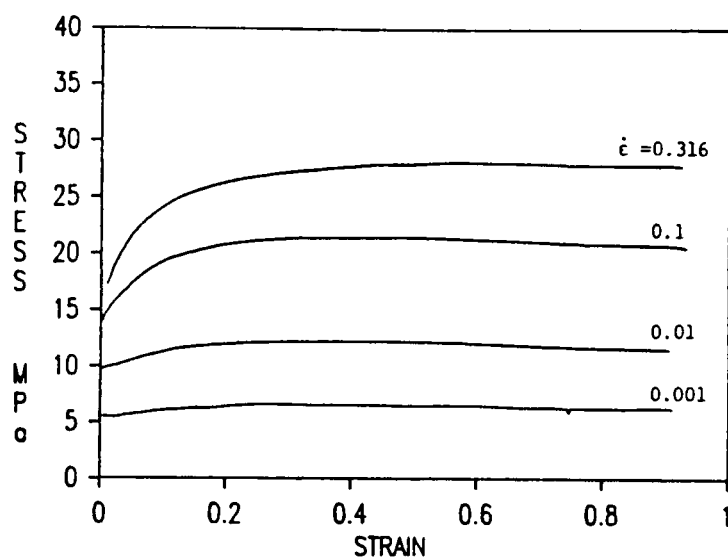


FIGURE 1. Constant True Strain Rate Tests

Fe - 2% Si

900 C - Jump Tests

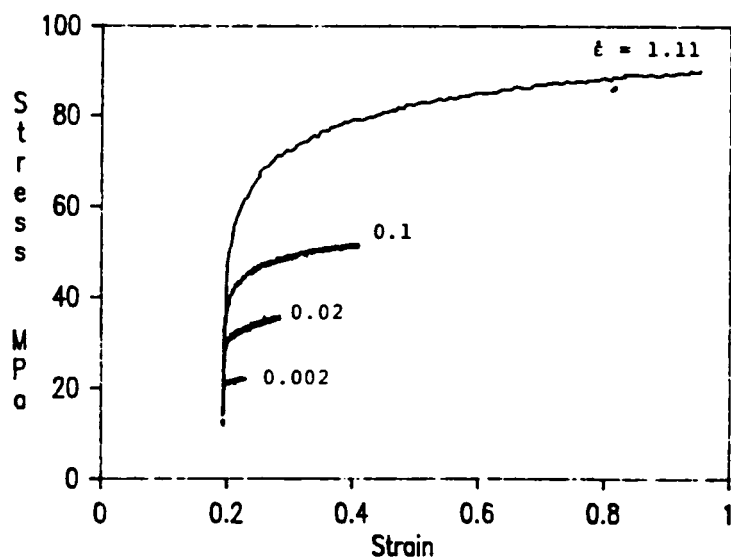


FIGURE 2. Strain Rate Jump Tests from the Same Initial Conditions.
(Initial Strain Rate = $2 \times 10^{-4} \text{ sec}^{-1}$)

Fe - 2% Si

1100 C

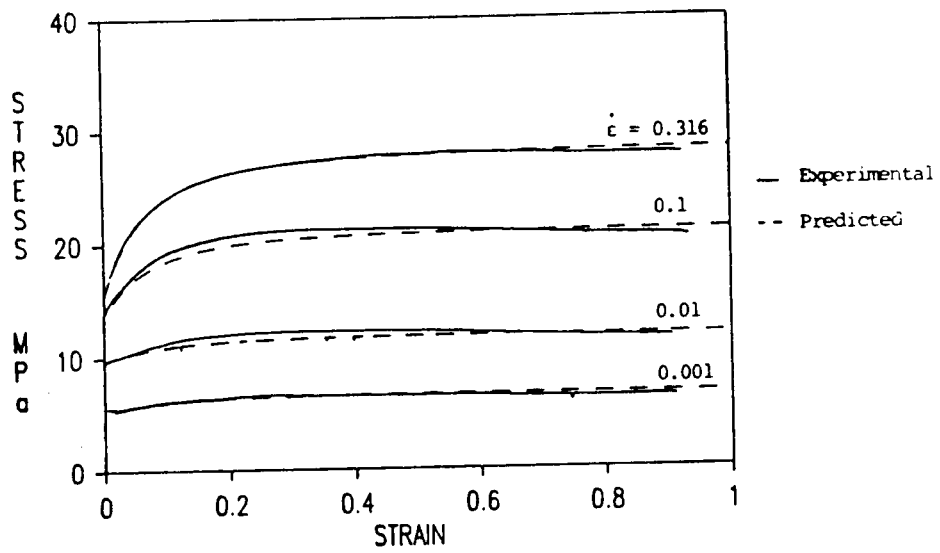


FIGURE 3. Predicted Stress/Strain Curves versus Experimental Data

Fe - 2% Si

900 C - Jump Tests

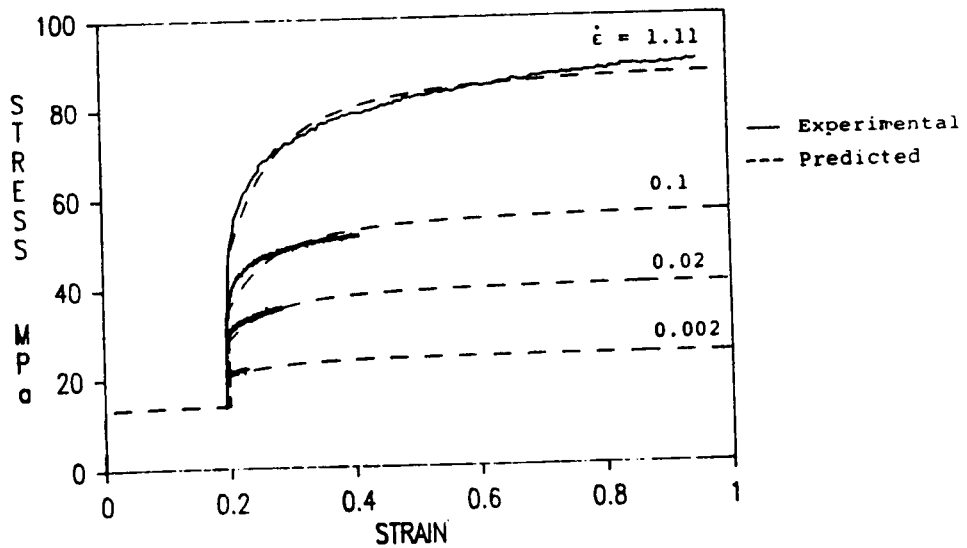


FIGURE 4. Predicted Strain Rate Jump Test, Stress/Strain Curves versus Experimental Data.